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Teachers' perception on the use of "Proof without Words (PWWs)" visualization of arithmetic sequences

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Teachers' perception on the use of "Proof without Words (PWWs)" visualization of arithmetic sequences

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Abstract. Teachers' perception or belief is seen as an important factor that influences teachers' teaching practices and students' learning. In this regard, this study aims to explore teachers' perceptions on the use of Proofs without Words (PWWs) for visualizing mathematics concepts. This research involves mathematics teachers with a diverse teaching experience. This study employed a qualitative approach with interview as data collecting technique. A total of 12 teachers were interviewed in a semi structured way. The questions that asked to them during interview were focused on how their perception saw about PWWs. The result of the interviews revealed that the PWWs can be used as a source of problems for students for problem-solving activities. However, it also found that students' mathematical knowledge and skills play an important role in process of interpreting the images.

1. Introduction

Mathematics is universal for which almost every other field of science utilizes mathematical concepts and principles. The objects studied in mathematics include numbers, patterns, algebra, geometry, calculus, and statistics. According to Descartes, mathematics is the science of order and measure[1]. Mathematics has abstract objects and its truth is based on logic. The objects of mathematics, which are mainly in the form of mathematical concepts, are mostly connected to each other and hierarchically structured. In terms of school mathematics, the focus of mathematics is not only mathematical concepts, but also mathematical skills. The National Council of Teachers of Mathematics (NCTM)[2] in its Principles and Standards for School Mathematics considered a number of mathematical skills as the fundamental aspects of mathematics. These mathematical skills include representation, connection, reasoning and proof, communication, and problem solving. With respect to reasoning and proof, NCTM stated that mathematics classrooms should enable students to develop and evaluate mathematical arguments and proofs and to select and use various types of reasoning and methods of proof.

The importance of proof as a fundamental aspect of mathematics should be considered in the learning of mathematics. Proof is not only the objective of mathematics learning, but also can be used as a tool to help students understand mathematical concept. With this respect, Alsina and Nelsen [3]offer an alternative way to do mathematical proof that is called as Proof without Words (PWWs). PWWs is a mathematical proof that uses image(s) as a prefix in visual thinking. PWWs are images or diagrams that help an individual to understand why a mathematical statement is true. After looking at

the images or diagrams, he or she can understand where to start to prove a mathematical statement. Essentially, PWWs provide visual guidance to someone, fortunately to stimulate the person thinking mathematically [4]. Nelsen and Alsina [3]also believe that PWWs can play a role in learning mathematics from elementary to university.

Images have a powerful benefit to illustrate or visualize mathematical concepts. In this regards, Livie and Lentz in Sanaky [5]presented four media functions of visual media, i.e. attention function, affective function, cognitive function and compensatory function. Attention function refers to the characteristic of visual media to grab the attention of students to concentrate on the content that is embedded in the visual media. Affective function of visual media can be seen from students' enjoyment level when learning to read pictorial text. Visual images or symbols will inspire students' emotions and attitudes. With regard to cognitive function, visual media reveals that visual symbols facilitate the achievement of goals to understand and hear information or messages contained in the picture. Lastly, the compensatory function means that visual media provides context to understand text helps weak students in reading to organize inform in text and remember. The importance of visualization in the learning of mathematics is also highlighted by Vilario Giardino. Vilario Giardino states that the mathematical visualizations and intuitions are interrelated. Mathematics intuition can be expressed as using the vision function but not using the eye. When a person is unable to visualize a proof then that's when he turns to intuition. Mathematical reasoning with images involves both visualization and intuition. Mathematics intuition relies heavily on the background of the person's knowledge and math skills, and it can determine the generalities of the conclusions obtained [6]. This is in line with Brating and Pejlare [7] referring to Giaquinto [8] who argue that visual thinking can be a helpful tool in discovery in geometry but only in limited cases in basic analysis. However, Brating and Pejlare argue that it is inappropriate to divide mathematics into 'visible' and 'not so visible' mathematics. It is necessary to consider what we want to visualize and to whom. For example, educated mathematicians have no problem communicating through visualization, as well as students who have understood the role of this visualization in mathematics. For people who are unfamiliar with relevant mathematical theories, this visualization can mean something quite different. Brating and Pejlare argue that mathematical visualization has no meaning independent of the visual observer itself, in the sense that the meaning of visualization depends on meaning by the observer itself

With regard to the use of images or visualization in the learning of mathematics, the ability and understanding of teachers in understanding the media and the image is very influential. Stylianou [9] said that students' ability to develop their conceptual understanding and ability to use a representation is influenced by the nature of the concept of teacher representation. Stylianou revealed that teachers may have gaps in their own perception and ability to use representation and visual media while performing and teaching mathematics. Therefore, this study is aimed to explore the perceptions of teachers on Proofs without Words (PWWs) images, and their implementation in mathematics learning.

2. Methods

This study was a descriptive qualitative. A total 10 senior high school mathematics teachers (R1, R2, R3, …, R10) were interviewed in a semi structured way[10]. The questions that asked to them during interview were focused on teachers' perception about PWWs. This perception covered 10 aspects related to PWWs that included: degree of urgency, degree of concept accuracy, degree of feasibility to understanding the concept, degree of interest, degree of variability, degree of basic abstraction, degree of reflective thinking, degree of inquiry, degree of integrated activity, and the simplicity of operating and creating [11]. In order to illustrate PWWs, the teachers were shown an example of PWWs for a case of arithmetic series during the interview.

Two PWWspictures(see Figure 1) were used to illustrate the concept of arithmetic sequences and its sum. Arithmetic sequences is sequences that can be write as a, $a + b$, $a + 2b$, $a + 3b$, … with a is the first element (U_1) of the sequence and b is the difference between the nearest element[12]. The sum of arithmetic sequences (S_n) is defined as $U_1 + U_2 + ... + U_n$.

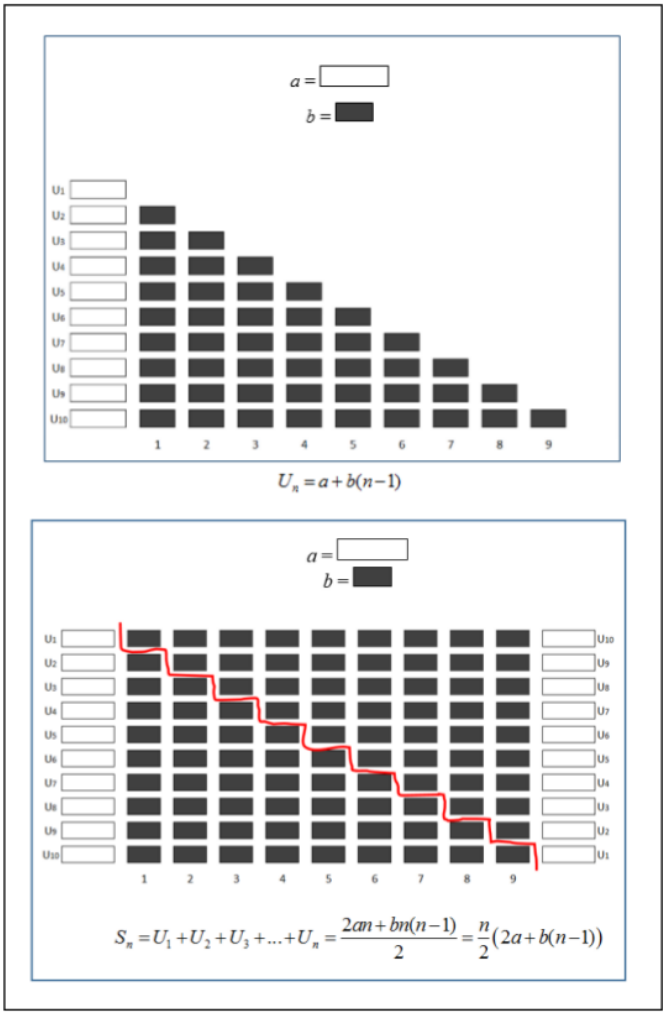

Figure1. PWWs Pictures for Arithmetic Sequences.

The data obtained by showing the image to the respondent, then they were given thirteen questions, the questions were:

Introduction Questions:

When did you start teach or working in the mathematics education field?

Have you ever heard "*Proof without Word (PWWs)*" in mathematics?

Pedagogical Aspects Questions

Degree of Urgency: What do you think about how important The PWWs role in help student learning about concept / idea mathematics destination compared if no using these media?

Degree of concept accuracy: What do you think about how accuracy the concept that is described or generated from the PWWs?

Degree of feasibility to understanding the concept: What do you think about PWWs' role in giving convenience and clarity for students for understanding concepts / mathematics ideas?

Degree of interest: Do you think PWWs will make student interest to learning mathematics? Why?

Degree of variability: What do you think about how flexibility of using PWWs in various model/approach/method in instructions?
Degree of basic for abstraction: Do you think the PWWs can assist students in abstracting the mathematical concepts contained in it? Why?
Degree of reflective thinking: Do you think that PWWs can stimulate students to perform reflection activities? Why?
Degree of inquiry: How do you think the role of the PWWs is in helping students discover the mathematical concepts / ideas they are targeting?
Degree of integrated activity: How do you think about the role of PWWs is in helping students to engage in integrated skills (thinking, speaking, and moving) in learning the mathematical concepts?

Practical Aspects Questions
How do you think the practicality of PWWs media when used in learning?
Do you think the PWWs media is easy to create?

The teachers' responses were analyzed in a descriptive qualitative way.

3. Result and Discussion

As stated earlier that Nelsen and Alsina[3] argued that PWWs have the potential to play an important role in mathematics learning, this study attempted to explore teachers' opinions and perceptions of PWWs in mathematics learning. Based on interview results, the information emerged were there are four teachers (R2, R6, R7, R10) had more than 10 years experience as teacher, four teachers (R1, R3, R4, R8) had less than 10 years experience as teacher, and the other two (R5, R9) had more than 20 years experience as teacher. Three teachers (R1, R5, and R7) stated that they had ever heard the terminology of PWWs, and the others stated that they never heard the terminology. It is shown that the PWWs is actually new for the majority of the teachers.

In the aspect of degree of urgency, all teachers stated that PWWs can play important role in mathematics learning activity, this statement is this is in line with Nelsen and Alsina [3] and Brating and Pejlare [7] statements. R2 said that the PWWs can be an alternate media than can be used in learning activity. In the aspect of degree of concept accuracy, nine teachers said that PWWs can describe the mathematics concept accurately, but R10 stated that he cannot feel or see the accuracy that PWWs may behave. Although R3 said that PWWs have the accuracy to describe the concept, but it would be not accurate enough if PWWs were used in the junior high schools. In the aspect of degree of feasibility to understanding the concept, nine teachers stated that PWWs would help the student to understand the mathematics concept, but R8 said that PWWs would not help directly the student to understand the mathematics concept, the need more explanation about the concept, and the PWWs did not provide explanation enough. These statements were in line with Giardino [6], Brating and Pejlare [7] statements that the benefit from using visualizations was depend on student prior knowledge and skills.

In the aspect of degree of interest, seven teachers said that PWWs is an interesting media, but R7 and R8 stated that PWWs might be interesting and might be not interesting, it was depend on students' characteristic. R10 stated that PWWs was not an interesting media, because there were a lot of media that more interesting than the PWWs. In the aspect of degree of variability, eight teachers said that that PWWs is can be used in various models of learning, but R2 and R3 stated the otherwise. R2 said that PWWs was not suitable for expository learning model. In the aspect of degree of basic abstraction, eight teachers said that PWWs can help student to do abstraction the concept, but R2 stated that the PWWs pictures were hard to understand, so he doubted that the PWWs will help in this aspect, so did R10 stated the same things.

In the aspect of degree of reflective thinking, five teachers (R2, R5, R6, R8, and R9) said that PWWs can help student to do reflective thinking and the other (R1, R3, R4, R7, R10) did not agree

with that. R1, R3, R4, R7, R10 said that PWWs was a new things, it was hard to understand, and it could not help the student to judge their own knowledge of the concept. In the aspect of degree of inquiry, all the teachers said that PWWs can help the student to construct their knowledge about the concept, even R1 stated that the PWWs would be meaningful to the student. But to do this, R8 said that the teacher have to be careful to guide student construct the knowledge. In the aspect of degree of integrated activity, only four teachers (R2, R5, R8, and R9) that said the PWWs can facilitate the student to do integrated activity, and the six others said that PWWs could not facilitate the student to do such activity. They said PWWs only facilitate student to do thinking activity.

In the aspect of the simplicity of operating, nine teachers said that the PWWs is easy to use, but R3 said that the PWWs is not easy to use because it was hard to understand. In the aspect of the simplicity of creating, five teachers (R5, R6, R7, R8, and R10) said that PWWs was easy to made, but the other teachers (R1, R2, R3, R4, and R9) said that it was easy to copy the pictures but it was very hard to create a new one with a new concept.

In general, interview results are summarized in the table below.

Table 1. Summaries of interview results.

Aspect	Positive Opinion	Negative Opinion	Percentage	
			Pos. Opinion	Neg. Opinion
Degree of Urgency	PWWs help in learning in learning activity, an it will have important role I learning		100	-
Degree of concept accuracy	PWWs are accurate in describing the concept	the accuracy of PWWs in describing the concept was not found	90	10
Degree of feasibility to understanding the concept	PWWs will help student to understand the concept	PWWs will not help student to understand the concept	90	10
Degree of interest	PWWs is interesting	PWWs is not interesting	70	30
Degree of variability	PWWs are flexible to a variety of learning models	PWWs are not flexible to a variety of learning models	80	20
Degree of basic for abstraction	PWWs will help students in abstracting activities	PWWs will not help students in abstracting activities	80	20
Degree of reflective thinking	PWWS helps students reflect	PWWS doesn't helps students reflect	50	50
Degree of inquiry	PWWs helps students discover the concept, PWWs in meaningful to student		100	-
Degree of integrated	PWWs help students in	PWWs doesn't help	40	60

Aspect	Positive Opinion	Negative Opinion	Percentage	
			Pos. Opinion	Neg. Opinion
activity	activities in an integrated way	students in activities in an integrated way, it only help in thinking activity		
The Simplicity of Operating	PWWs are practical to use	PWWs are not practical to use	90	10
The Simplicity of creating	PWWs is easy to be made	PWWs is not easy to be made	50	50

Based on the table, the most powerful aspects of PWWs are aspects of the degree of urgency and degree of inquiry in which all respondents gave positive opinions about the PWWs. In pointing out the power of PWWs on the aspect of the degree of urgency respondent R9 stated: "Although I just heard the term PWWs, but after observing to the two images above I can argue that PWWs are very important for the constructing of mathematical concepts for students, because with the help of PWWS concepts or mathematical ideas can be visualized so that students see the concept / idea directly", while in pointing out the PWWs power on the aspect of degree of inquiry urgency respondent R1 stated: "The images in PWWs contain the meaning that guides and directs the students to find a mathematical concept". However, there were three weak aspects of this PWWs. These aspects were the degree of reflective thinking, the degree of integrated activity and the simplicity of creating, in which over 40% of respondents gave negative opinions. To show the weakness of PWWs in the aspect of degree of reflective thinking, Respondent R1 stated "Students cannot judge their own mathematics skills only by their ability to understand PWWs", while to show the weakness of PWWs in the degree of integrated activity aspect R1 stated: " PWWs may be helpful in developing thinking skills, but not in communicating and moving activity"; while to show the weakness of PWWs in the simplicity of creating aspect, Respondent R9 stated: "I think PWWs media is quite easy to create, but it is difficult to find ideas in visualizing abstract concepts for us represent in the PWWs form".

4. Conclucions

Basedon the results of interviews analysis of the 10 teachers, we concluded that the term media PWWs is a relatively new and of the ten teachers only three of them has ever heard it. From the interview result we can see that teacher with experience teaching more than 20 years (R5 and R9) have positive perceptions towards PWWs. The majority of teachers believe that PWW has powerful strength in the aspect degree of urgency and degree of inquiry. However, there was a There are some negative opinions about PWWs especially on aspects degree of reflective thinking and degree of integrated activity from teachers who have less than 20 years of work experience. Overall, most of the teachers here, have positive perception about PWWs, except on the degree of reflective thinking and the simplicity of creating. Their statement answer Nelsen and Alsina' [3]faith that can also play a role in learning mathematics but in its utilization need to be considered also the character and the experience of the students because math reasoning with images involves both visualization and intuition. Mathematics intuition relies heavily on the background of the person's knowledge and math skills, and it can determine the generalities of the conclusions obtained[6]. PWWs is easy to use in mathematics learning, but it is very hard to create a new PWWs pictures from a new concept.

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